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THE APPLICATION OF ANGULAR VARIABLE TECHNIQUE TO PLASMA POLARIMETRY

Key words: plasma, polarimetry, AVT.

Abstract: An efficient theory of the polarization state evolution of electromagnetic wave at the magnetized plasma is presented. Angular variable technique (AVT), developed on the basis of quasi-isotropic approximation (QIA) of geometrical optics, describes both quasi-longitudinal propagation (Faraday Effect) and quasi-transverse propagation (Cotton-Mouton effect), even in the case when both effect combine nonlinearly. It could be used as a theoretical background for a polarimetric diagnostic of at any type laboratory or interstellar plasma as long as the conditions of weak anisotropy and weak inhomogeneity are fulfilled. As an example, equations of AVT are applied to magnetized plasma with parameters typical for magnetic fusion devices.

Zastosowanie metody zamiennych kątowych w polarymetrii plazmy

Słowa kluczowe: plazma, polarymetria, AVT.

Streszczenie: W artykule przedstawiona jest teoria ewolucji stanu polaryzacji fali elektromagnetycznej w niejednorodnym ośrodku, jakim jest plazma znajdująca się w polu magnetycznym. Prezentowana technika zmiennych kątowych (AVT) została opracowana na podstawie przybliżenia quasi-izotropowego (QIA) optyki geometrycznej. Opisuje ona zmiany polaryzacji zarówno w przypadku propagacji równoległej do pola magnetycznego (efekt Faradaya), jak i propagacji poprzecznej (efekt Cotton--Moutona). Umożliwia również interpretację pomiaru polarymetrycznego, w przypadku gdy oba efekty są istotne i oddziałują ze sobą nieliniowo. Proponowany formalizm może stanowić podstawę teoretyczną diagnostyki polarymetrycznej dla dowolnego typu plazmy, o ile spełnione są warunki słabej anizotropii i słabej niejednorodności. Równania AVT zastosowano do namagnesowanej plazmy o parametrach występujących we współczesnych reaktorach termojądrowych.

Introduction

It is well known fact that more than 99,9% of the matter in the universe is in the form of plasma, which is the fourth state of matter after solid, liquid, and gas. The natural sources of plasma are the stars, atmosphere, lightings, and gaseous nebula. Nevertheless, various types of plasma are created in laboratories for different applications, which include arcs, gaseous discharge, laser produced plasma, and recently the most importantly, tokamak plasma. These plasma sources might have various applications in different fields of research and industry. Typical plasma parameters cover dozens orders of magnitude: size -10^{-6} m (lab plasma) -10^{25} m (intergalactic nebula), density $-1m^{-3}$ (intergalactic medium) -10^{35} m⁻³ (white dwarfs), temperature $-\sim 0$ K (intergalactic plasma) -10^{9} K (fusion plasma) (Fig. 1), lifetime -10^{-12} s (laser-produced plasma) -10^{17} s (intergalactic plasma), magnetic field -10^{-4} T (lab plasma) -10^{11} T (near neutron stars).

There are many methods for studying such a diverse medium, but one of the most important is polarimetry. This method is based on the fact that for high-frequency electromagnetic waves plasma, in the presence of a magnetic field, becomes a birefringent and optically active medium with strong dependence on plasma density and magnetic field. As a consequence

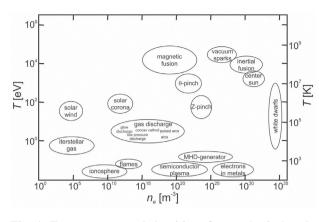


Fig. 1. Temperatures and densities of astrophysical and laboratory plasma [1]

Source: Authors based on [1].

the measurement of changes in the wave polarization during its propagation into plasma, it provides valuable information on plasma density and magnetic field. Especially the last one is very important, as polarimetry is one of the few techniques to measure the magnetic field in the interior of the plasma. Usually, the change in the polarization state is considered in two separate cases [2, 3]. The first one take place when propagation direction is parallel to the magnetic field, so plasma is optically active and imposes the Faraday Effect (F). The polarization ellipse (and, for linear polarization, the polarization plane) rotates, with a constant ellipticity, so only its orientation changes (Fig. 2a). The second one is for propagation perpendicular to the magnetic field, so plasma is purely birefringent and imposes the Cotton-Mouton effect (CM). In this case, the ellipticity of the polarization ellipse is changing with a constant polarization plane (Fig. 2b).

a)

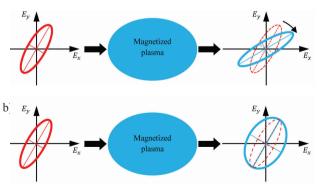


Fig. 2. The polarization ellipse change in Faraday (a) and Cotton-Mouton (b) effect

Source: Authors.

The coupling between both effects appears when the magnetic field has both parallel and perpendicular components with respect to the wave propagation direction. It complicates the interpretation of polarimetry measurements, especially when F and CM effects are strong [2, 3] and more sophisticated treatment is indispensable. Here, we present the theory of electromagnetic wave polarisation state evolution in heterogeneous magnetized plasma for any coupling between F and CM effect. The only restriction is weak anisotropy and weak inhomogeneity of the analysed medium.

The paper is organised as follows: Section 2 outlines the basic equations of quasi-isotropic approximation and derives AVT equations in (ψ , δ) from QIA equations. Section 3 rewrites AVT equations in the case of cold plasma approximation. Section 4 analyses the choice of the appropriate wavelength of the beam applied at polarimetric diagnostic with the use of an example of modern tokamak plasma conditions.

1. Polarization state evolution

It is commonly accepted to characterize an electromagnetic wave traveling along the *z* axis as the vector sum of two harmonic electric fields $\mathbf{E}_{x}(z,t)$ and $\mathbf{E}_{xy}(z,t)$, whose directions are parallel to the *x* and *y* axes respectively:

$$\mathbf{E}(z,t) = \mathbf{E}_{x}(z,t) + \mathbf{E}_{y}(z,t) =$$

$$= E_{x0}e^{i(\omega t - kz + \delta_{x})}\mathbf{e}_{x} + E_{y0}e^{i(\omega t - kz + \delta_{y})}\mathbf{e}_{y} =$$

$$= \left(E_{x0}e^{i\delta_{x}}\mathbf{e}_{x} + E_{y0}e^{i\delta_{y}}\mathbf{e}_{y}\right)e^{i(\omega t - kz)} = \Gamma e^{i(\omega t - kz)}$$
(1)

where \mathbf{e}_x and \mathbf{e}_y are unit vectors along the coordinate axis. Γ is the polarization vector

$$\Gamma = E_{x}e_{x} + E_{y}e_{y} = E_{x0}e^{i\delta_{x}}e_{x} + E_{y0}e^{i\delta_{y}}e_{y}$$
(2)

orthogonal to the beam propagation direction and dependent on the magnitudes E_0 and phases δ of both complex components E_x and E_y . For the beam passing through anisotropic medium, as plasma located at external magnetic field, its polarization state changes along the path. An adequate method for the description of such an evolution is the Budden's method [4, 5], and its extension – the quasi-isotropic approximation (QIA) [6, 7]. Both methods deal with coupled wave equations for the components of the electromagnetic wave field. In the case of the quasi-isotropic approximation of the geometrical optics method, basic assumptions are that the scale of medium inhomogeneity L is much larger than the beam wavelength λ

$$\lambda \ll L$$
 (3)

(weak inhomogeneity condition) and that the full tensor of electrical permittivity of an anisotropic medium ε_{mn} could be divided into two parts: the electrical permittivity ε_0 of the isotropic background medium and the anisotropy tensor v_{mn}

$$\varepsilon_{\rm mn} = \varepsilon_0 \delta_{\rm mn} + \nu_{\rm mn} \tag{4}$$

with components much smaller than ε_0

$$\max[v_{mn}] \ll \varepsilon_0 \tag{5}$$

(weak anisotropy condition). According to [6,7], an asymptotic solution to Maxwell's equation in small parameters $\mu_{\rm G} = \lambda / L$ and $\mu_{\rm A} = \max(\lfloor v_{\rm mn} \rfloor / \varepsilon_0)$ leads to the set of complex ordinary equations for the components $E_{\rm x}$ and $E_{\rm y}$ of the polarization vector:

$$\begin{cases} \frac{dE_x}{dz} = \frac{1}{2} i k_0 \varepsilon_0^{-1/2} \left(v_{11} E_x + v_{12} E_y \right) \\ \frac{dE_y}{dz} = \frac{1}{2} i k_0 \varepsilon_0^{-1/2} \left(v_{21} E_x + v_{22} E_y \right) \end{cases}$$
(6)

where k_0 is the local wave number of the electromagnetic beam.

Although the equations (6) fully describe the evolution of the polarization vector along the trajectory, they are not used in classical polarimetry. The reason is in fact that the equations (6) describe the evolution of both complex components E_x and E_y and therefore of all four parameters, E_{x0} , E_{y0} , δ_{x2} and δ_y of the polarization vector. In contrast, polarimetric systems usually measure only the amplitude ratio E_{y0}/E_{x0} and phase difference $\delta_y - \delta_x$ between two components of the polarization vector and present them as Stokes vector components [8] or any other equivalent quantities describing polarization state, like complex amplitude ratio [9], complex polarization angle [10] or pair of any two angular parameters of polarization ellipse [8] (Fig. 3).

It becomes necessary to transform the equations (6) to the form representing the evolution of measurable parameters, e.g., the set of azimuthal angle ψ and the phase difference angle δ . According to [8, 11], between angular variables set (ψ , δ) and polarization vector components E_x and E_y , there are the following relations:

$$\begin{cases} \psi = \frac{1}{2} \arctan\left(\frac{E_x E_y^* + E_x^* E_y}{E_x E_x^* + E_x^* E_y}\right) \\ \delta = \arctan\left(\frac{i\left(E_x E_y^* - E_x^* E_y\right)}{E_x E_y^* + E_x^* E_y}\right) \end{cases}$$
(7)

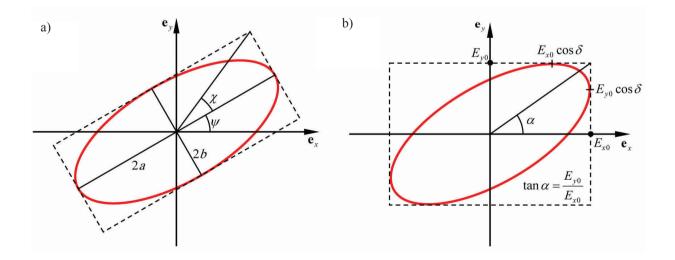


Fig. 3. Angular parameters of polarization ellipse: a) azimuthal angle ψ and ellipticity angle χ ; b) amplitude ratio α and phase difference δ

where the overbar denotes a scalar complex conjugate. Taking derivatives of both equations (7) and substituting proper relations from (6), one can obtain that angular variables set (ψ , δ) which obeys equations:

$$\begin{cases} \frac{d\psi}{dz} = \frac{1}{2} \left(A_{1} \sin(2\psi) - A_{2} \cos(2\psi) \right) \sqrt{1 + \left(\sin(2\psi) \tan(\delta) \right)^{2}} \\ + \frac{1}{2} \left(\Omega_{3} - \left(\Omega_{1} \cos(2\psi) + \Omega_{2} \sin(2\psi) \right) \sin(2\psi) \tan(\delta) \right) \\ \frac{d\delta}{dz} = \left(A_{2} \sin(\delta) - A_{3} \cos(\delta) \right) \sqrt{1 + \left(\frac{\cos(\delta)}{\tan(2\psi)} \right)^{2}} \\ + \Omega_{1} - \left(\Omega_{2} \cos(\delta) + \Omega_{3} \sin(\delta) \right) \frac{\cos(\delta)}{\tan(2\psi)} \end{cases}$$
(8)

where

$$A_{1} = Im\left(\frac{1}{2}k_{0}\varepsilon_{0}^{-1/2}(v_{11}-v_{22})\right)$$

$$A_{2} = Im\left(\frac{1}{2}k_{0}\varepsilon_{0}^{-1/2}(v_{12}+v_{21})\right)$$

$$A_{3} = Im\left(-\frac{1}{2}ik_{0}\varepsilon_{0}^{-1/2}(v_{12}-v_{21})\right)$$
(9)

are components responsible for dichroism and

$$\left(\begin{array}{l} \Omega_{1} = \operatorname{Re}\left(\frac{1}{2}k_{0}\varepsilon_{0}^{-1/2}\left(v_{11} - v_{22}\right)\right) \\ \Omega_{2} = \operatorname{Re}\left(\frac{1}{2}k_{0}\varepsilon_{0}^{-1/2}\left(v_{12} + v_{21}\right)\right) \\ \Omega_{3} = \operatorname{Re}\left(-\frac{1}{2}ik_{0}\varepsilon_{0}^{-1/2}\left(v_{12} - v_{21}\right)\right) \end{array}$$
(10)

are components responsible for birefringence. The obtained equations could be used to the analysis of polarimetric measurement for any type of the plasma, as long as the conditions of weak inhomogeneity (3) and weak anisotropy are fulfilled (5). Moreover, in the most common case of a non-adsorbing medium, like collisionless plasma, $A_i = 0$ and equations (10) simplify to

$$\begin{cases} \frac{d\psi}{dz} = \frac{1}{2} \left(\Omega_3 - \left(\Omega_1 \cos(2\psi) + \Omega_2 \sin(2\psi) \right) \sin(2\psi) \tan(\delta) \right) \\ \frac{d\delta}{dz} = \Omega_1 - \left(\Omega_2 \cos(\delta) + \Omega_3 \sin(\delta) \right) \cos(\delta) \tan^{-1}(2\psi) \end{cases}$$
(11)

2. Polarization state evolution in cold magnetized plasma

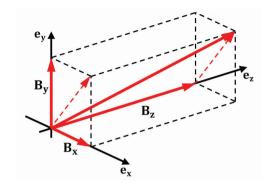


Fig. 4. Position of the external magnetic field B and unit vectors e_x and e_y relative the ray propagation direction e_y

Source: Authors.

In this section, we apply equations (8) to the analysis of the angular variable set (ψ, δ) evolution in weakly anisotropic, collisionless, plasma. In a coordinate system presented in Fig. 4, where the magnetic field has longitudinal component B_z and two transverse components B_x and B_y , the electrical permittivity ε_0 of the isotropic background medium is equal

$$\varepsilon_0 = 1 - v \tag{12}$$

and the anisotropy tensor v has a form

$$\boldsymbol{v} = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} = \frac{v}{1-u}$$
$$\begin{bmatrix} -\overline{u} \left(B_y^2 + B_x^2 \right) & i\sqrt{\overline{u}} B_z + \overline{u} B_x B_y \\ -i\sqrt{\overline{u}} B_z + \overline{u} B_x B_y & -\overline{u} \left(B_x^2 + B_z^2 \right) \end{bmatrix}$$
(13)

where

$$\overline{u} = \left(\frac{e}{m_e c\omega}\right)^2; \ u = \left(\frac{\omega_c}{\omega}\right)^2 = \left(\frac{eB_0}{m_e c\omega}\right)^2$$

$$v = \left(\frac{\omega_p}{\omega}\right)^2 = \frac{4\pi e^2 N_e}{m_e \omega^2}$$
(14)

are standard plasma parameters connected with plasma frequency ω_p , cyclotron frequency ω_c and the beam frequency $\omega = 2\pi c/\lambda$. Weak anisotropy, condition (5), requires

$$v \ll 1 \text{ and } u \ll 1 \tag{15}$$

or equivalently

$$\omega \gg \omega_p$$
 and $\omega \gg \omega_c$ (16)

which are guaranteed by the choice of appropriate short wavelength of the electromagnetic beam. The conditions (16) also ensure that there is no total reflection of an electromagnetic wave at a cutoff layer, where the local refractive index goes to zero.

Introducing (12)–(14) into (10), we find the values of vector Ω :

$$\Omega = \begin{bmatrix} \Omega_{1} \\ \Omega_{2} \\ \Omega_{3} \end{bmatrix} = \begin{bmatrix} \frac{e^{4}}{16\pi^{3}\varepsilon_{0}m_{e}^{3}c^{4}}\lambda^{3}\left(B_{x}^{2}-B_{y}^{2}\right)N_{e} \\ \frac{e^{4}}{16\pi^{3}\varepsilon_{0}m_{e}^{3}c^{4}}\lambda^{3}\left(2B_{x}B_{y}\right)N_{e} \\ \frac{e^{3}}{4\pi^{2}\varepsilon_{0}m_{e}^{2}c^{3}}\lambda^{2}B_{z}N_{e} \end{bmatrix} = \begin{bmatrix} 2.46\cdot10^{-11}\lambda^{3}\left(B_{x}^{2}-B_{y}^{2}\right)N_{e} \\ 2.46\cdot10^{-11}\lambda^{3}\left(2B_{x}B_{y}\right)N_{e} \\ 5.26\cdot10^{-13}\lambda^{2}B_{z}N_{e} \end{bmatrix}$$
(17)

frequently used in plasma polarimetry [12]. The first two components of vector Ω define the Cotton-Mouton effect, which depends on two components of external magnetic field perpendicular to the beam propagation direction. The third component defines the Faraday Effect, proportional to the component of external magnetic field parallel to the beam propagation direction. In the case of the pure Faraday Effect, when propagation is quasi-parallel, equations (11) simplify to

$$\begin{cases} \frac{d\Psi}{dz} = \frac{1}{2}\Omega_3 \\ \frac{d\delta}{dz} = 0 \end{cases}$$
(18)

So the evolution of the polarization ellipse is reduced to the rotation of its polarization plane at the Faraday angle

$$\Delta \Psi = 2.63 \cdot 10^{-13} \lambda^2 \int B_{\parallel} N_e \mathrm{d}z \tag{19}$$

In the same way, for pure a Cotton-Mouton Effect in the case of quasi-perpendicular propagation, the polarization state evolution is in the change of its phase difference angle:

$$\Delta \delta = 2.46 \cdot 10^{-11} \lambda^3 \int B_{\perp}^2 N_e \mathrm{d}z \tag{20}$$

In both cases, polarimetric measurement could be used to obtain information on the line integrated plasma density when the magnetic field value is known or vice versa as a magnetic field detector for given plasma density profile.

In general case, when the magnetic field has both parallel and perpendicular component, both effects start to combine nonlinearly. In such a case, to obtain the information on plasma density or the magnetic field value the system, (11) has to be solved numerically by a sophisticated method of polarimetric data inversion [13, 14]. The associated calculations are very time consuming, and the analysis of polarimetric measurement is very hard. It is better to design the polarimetric system in such a way that only one effect takes place. Another option is to use the electromagnetic beam with plane polarization ($\delta = 0$) of the azimuth angle $\psi = \pi/4$ and the proper wavelength value, so Faraday rotation and Cotton-Mouton phase shift are low: $\Delta \psi <$ 0.5 rad and $\Delta\delta < 0.5$ rad. Interest in this case stems from fact that, along the whole path, $\sin(2\psi) \cong \cos(\delta) \cong 1$ and $\tan(\delta) \cong \tan^{-1}(2\psi) \cong 0$. Under these conditions, system (11) could be written as

$$\begin{cases} \frac{d\psi}{dz} \approx \frac{1}{2}\Omega_3 \\ \frac{d\delta}{dz} \approx \Omega_1 \end{cases}$$
(21)

Therefore, both Equations (19) and (20) are correct, and there is only a second order coupling between Faraday and Cotton-Mouton Effects and the analysis is greatly simplified.

3. Probing beam wavelength

One of the most important tasks in the design of a polarimetric systems is the selection of proper probing beam wavelength, matched to the length *L*, density N_e and magnetic field **B** of the plasma to be tested. There are several constrains to be taken into account. Some of them were discussed at the article. The requirement of weak anisotropy, set by conditions (16), limits the maximum wavelength from the top. The limit can be taken as follows:

1) $\omega > 0.1\omega_{n}$,

2)
$$\omega > 0.1\omega'$$
.

Another restriction on the maximum wavelength comes from the fact that, for some λ value, the change in the azimuthal angle ψ and the phase difference angle δ exceed 2π and the measurement becomes ambiguously. To avoid such a situation, the F and CM Effects have to be smaller than 2π as follows:

3)
$$\Delta \psi < 2\pi$$
,
4) $\Delta \delta < 2\pi$.

On the other hand, the effect of polarization change has to be measurable with sufficiently high signal to noise ratio. As the Faraday rotation is proportional to the λ^2 and phase shift to λ^3 , the minimum value of a wavelength is constraint. It is reasonable to assume that measured value of Faraday rotation angle and Cotton-Mouton phase shift has to be at least $0.1^{\circ} \cong 0.002$ rad: 5) $\Delta \psi > 0.002$,

6) $\Delta \delta > 0.002$.

It has to be pointed out that mechanics, optics, and electronic (e.g., minimum disturbance by vibrations and refraction, detectors sensitivity) also force specific restrictions for the beam wavelength, but this is beyond the scope of the present article.

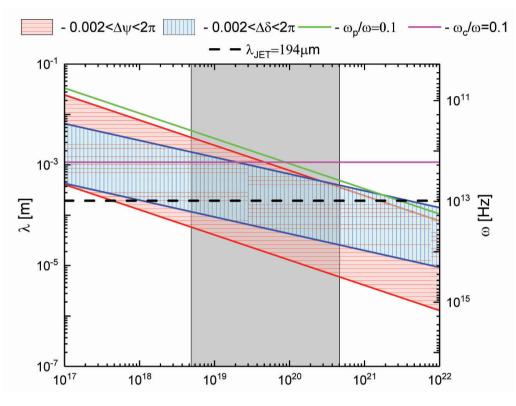


Fig. 5. Constrains for polarimetric wavelength at tokamak JET plasma conditions

Source: Authors.

Figure 5 presents such an analysis for plasma and magnetic field conditions typical for JET - the biggest in the world thermonuclear experimental reactor. At Jet the beam passes through the plasma on the distance $L \cong 2m$, plasma density varies during the experimental pulse within $N_e \simeq (5 \cdot 10^{18} - 5 \cdot 10^{20}) \,\mathrm{m}^{-3}$ range, perpendicular magnetic field is $B_{\perp} \simeq 3T$, and parallel magnetic field is $B_{\parallel} \cong 0.2$ T. The existing JET polarimetric diagnostic system relies on using far infrared laser (terahertz region in frequency domain) with a wavelength of 195 µm (Deuterated cyanide (DCN) laser). As it is clearly seen in Fig. 5, the applied wavelength allows the measurement in the assumed sensitivity limits for the whole plasma densities conditions. However, for densities $N_e > 5 \cdot 10^{19} \,\mathrm{m}^{-3}$, both Faraday rotation and Cotton-Mouton phase shift exceed 0.5 rad and coupling between both effects is strong. For this reason, decreasing the wavelength to 119 µm (methanol laser) seems to be advisable; although, for low plasma densities, only the Faraday Effect is measurable, but at medium and high densities, the F and CM would be in the range where coupling between them is negligible and the analysis is straightforward.

Summary

Differential equations for evolution of angular variables set (ψ, δ) are derived on the basis of quasi isotropic approximation of the geometric optics method. These equations can be integrated numerically for arbitrary profiles of the electron density and for arbitrary configurations of the static magnetic field. The equations for AVT admit exact analytical solutions in the case of pure Faraday and pure Cotton-Mouton Effects and approximate solutions by the polarimetric data inversion method or by the perturbation method in the case of the strong coupling between both effects. Besides, equations

for evolution could be applied to any plasma type with parallel and perpendicular gradients of plasma density and magnetic field components, as long as plasma anisotropy and inhomogeneity are weak.

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